ProAdvice 3: AILERON SIZING

Introduction

The purpose of the ailerons is to provide control about the airplane’s roll axis. There are three common types of ailerons used in modern airplanes; Plain Flap Ailerons, Frise Ailerons, and Spoiler-Flap Ailerons. Schematics of these are shown in Figure 1. Other aileron types include the Flaperon (a combination of flaps and ailerons) and Elevon (a combination of elevators and ailerons).

Figure 1: Three common types of ailerons; Plain Flap (top), Frise (middle), and Spoiler-Flap (bottom).

Plain Flap Ailerons
The plain flap is the most common type of aileron configuration. They are very effective and inexpensive to manufacture. For this reason, they can be found on a wide range of aircraft, ranging from primary trainers to commercial aircraft. As can be seen in Figure 1 the aileron on the up-going wing is deflected Trailing Edge Down (TED) and the down-going wing is deflected Trailing Edge Up (TEU).

Frise Ailerons
The Frise aileron was invented by the famed designer Leslie George Frise BSc FRAeS (1897-1979)¹. Their purpose is to reduce or eliminate adverse yaw (see Section 22), but also to reduce hinge moments. This can be accomplished in several ways, of which one is shown in Figure 1. All require the hinge to be offset to the lower surface as shown in the figure.

The geometry of the aileron forces the leading edge of the aileron that is deflected Trailing Edge Up (TEU) downward and outside of the regular Outside Mold Line (OML). This exposes it to the airstream and increases the drag on that side of the wing (the down-going side). The drag generates a yawing moment and reduces the

¹ Among well know aircraft whose design he contributed to in are the Bristol Fighter, Bristol Bulldog, Bristol Beaufighter, and the Percival Jet Provost.
tendency of the wing to yaw “out of the turn”, opposite the bank. If the leading edge of the aileron is round like the one shown in the figure, a powerful low pressure region is generated that lowers the hinge moment. This explains its use in both fast and large aircraft before the advent of hydraulically boosted control systems. If the nose is too sharp the lower surface may stall, which can cause severe buffeting (Reference 1). This type of aileron has seen use on many different aircraft types; among them the B-17, Bell P-39, Grumman F6F-3 and TBF, the Spitfire, Hurricane, Focke-Wulf 190, Curtiss Wright C-46, and DC-4, and many Cessna models.

**Spoiler-Flap Ailerons**

Several airplanes feature this type of ailerons (e.g. Mitsubishi MU-2, Boeing B-52, and others). This aileron type usually features a flap deflected Trailing Edge Down (TED) on the up-going wing and a small spoiler on the down-going side. As the spoiler is deployed it reduces lift on the down-going side. The aileron on that side may or may not deflect TEU at the same time. The spoiler increases the drag on the down-going wing and reduces adverse yaw. However, a common complaint is that such aileron systems tend to be sluggish at low airspeeds, as separated flow creeps forward toward the leading edge of the wing, and reduces the effectiveness of the spoiler. This control system may display peculiar side-effects on swept wing aircraft. As an example, it is well known that B-52 pilots complain about a significant nose pitch-up moment associated with aileron deflection. It turns out that as the spoilers are deployed the center of lift moves forward; destabilizing the aircraft such it pitches nose-up. An assertive nose pitch-down correction is required by the pilot.

**Aileron Design Requirements**

Among critical design points for ailerons are the following:

1. Responsiveness at slow speeds with large deflections.
2. Responsiveness at high speeds with low deflections.
3. Comfortable stick forces throughout flight envelope.

Another term for responsiveness is roll authority. Although responsiveness at slow speeds is imperative (low dynamic pressure requires greater deflection or control surface area, or a combination of the two), high speed functionality is of great importance as well. It has been known for a long time that a pilot’s conception of adequate roll control is tied to the helix angle made by the wing as the airplane rolls at a given airspeed\(^2\); \( \frac{pb}{2V} \). In this expression, \( p \) is the roll rate in radians per second for full aileron deflection, \( b \) is the wing span (in ft or m), and \( V \) is the airspeed (in ft/s or m/s). This way, it is recommended that for specific types of aircraft the following ratios are met or exceeded:

Cargo or heavy-lift aircraft: \[ \frac{pb}{2V} > 0.07 \]

Fighter aircraft: \[ \frac{pb}{2V} > 0.09 \]

For the aircraft designer this means that the physical dimensions of the ailerons can be determined based on the desired roll rate. A step-by-step design approach is presented below.

**Estimating Steady-State Roll Rate**

When the ailerons are deflected the airplane begins to roll. The motion consists of a transient acceleration that disappears once the roll rate increases and after which there is a steady roll rate. The steady state roll rate which is helpful in determining a pilot desired responsiveness.

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\(^2\) Per Airplane Performance Stability and Control by Perkins and Hage, page 352.
The steady state roll rate is determined from:

\[ \frac{pb}{2V} = -\frac{C_{ls_a}}{C_{lp}} \delta_a \]  \hspace{1cm} (1)

Note that the steady-state roll rate can be determined from:

\[ p = -\frac{C_{ls_a}}{C_{lp}} \delta_a \left( \frac{2V}{b} \right) \]  \hspace{1cm} (2)

Roll authority and roll damping for two special but common wing planform shapes.

**CASE 1: Straight Tapered Wing with Taper Ratio \( \lambda \):**

Roll authority:

\[ C_{ls_a} = \frac{c_{ls_a} C_R}{Sb} \left[ (b_2^2 - b_1^2) + \frac{4(\lambda - 1)}{3b} (b_2^3 - b_1^3) \right] \]  \hspace{1cm} (3)

Roll damping:

\[ C_{lp} = -\frac{(c_a + c_{do})}{24S} \frac{C_R b_1}{1 + 3\lambda} \]  \hspace{1cm} (4)

Units for both: **per radian** or **per degree**

**CASE 2: Rectangular Wing \( \lambda = 1 \):**

Roll authority:

\[ C_{ls_a} = \frac{c_{ls_a} (b_2^2 - b_1^2)}{b^2} \]  \hspace{1cm} (5)

Roll damping:

\[ C_{lp} = -\frac{c_a + c_{do}}{6} \]  \hspace{1cm} (6)

**DERIVATION:**

Assume the wing is rigid and the rolling motion is caused by deflecting the ailerons to an angle \( \delta_a \). Further assume the roll rate \( p \) is impeded by the roll damping due to a local change in AOA along the wing (with a minor contribution from the vertical tail). Therefore we can write the equation of rolling motion for the aircraft as follows:
\[ I_{XX} \dot{\delta} = \frac{\partial L}{\partial p} \left( \frac{pb}{2V} \right) + \frac{\partial L}{\partial \delta_a} \delta_a \]  

(i)

Where \( I_{XX} \) is the aircraft’s moment of inertia (in slugs·ft\(^2\) or kg·m\(^2\)), \( \dot{\delta} \) is the roll acceleration in rad/s\(^2\), \( L \) is the rolling moment in ft·lb or N·m, and \( \delta_a \) is the aileron deflection in degrees.

\[
0 = \frac{\partial L}{\partial p} \left( \frac{pb}{2V} \right) + \frac{\partial L}{\partial \delta_a} \delta_a \quad \Leftrightarrow \quad \frac{\partial L}{\partial p} \left( \frac{pb}{2V} \right) = -\frac{\partial L}{\partial \delta_a} \delta_a \quad \Rightarrow \quad \left( \frac{pb}{2V} \right) = -\frac{\partial L/\partial \delta_a}{\partial L/\partial p} \delta_a
\]

In terms of stability derivatives we can write:

\[
\left( \frac{pb}{2V} \right) = -\frac{C_{\delta a}}{C_{l p}} \delta_a
\]

(ii)

So, the problem boils down to the determination of the two derivatives \( C_{\delta a} \) and \( C_{l p} \), but this will be shown in a future ProAdvice.

QED

**STEP-BY-STEP: Aileron Sizing**

**STEP 1:** Establish initial dimensions based on Figure 3. Also determine the likely aileron deflection angle, \( \delta_a \). Note that the control system will stretch in flight reducing the maximum ground deflection. This means that a control system designed for a maximum deflection of, say, 15° on the ground, may only deflect as much as 75% of that in flight. Some control systems are so poorly designed\(^3\) that they may only achieve 25% of the maximum deflection. At any rate, 75% is a reasonable “first stab” estimate for an average control system. This would mean that a maximum deflection of 15° is closer to 11.3° in flight.

**STEP 2:** Using the geometry from STEP 1, estimate roll damping, \( C_{l p} \). Use Equation (4) for a straight tapered wing, and Equation (6) for a “Hershey bar” wing.

**STEP 3:** Using the geometry from STEP 1, estimate the roll authority, \( C_{l \delta a} \). Use Equation (3) for a straight tapered wing, and Equation (5) for a “Hershey bar” wing.

**STEP 4:** Determine a desired “target” roll helix angle per Section 23.5.2 using Equation (1). If the calculated value is less than the selected target enlarge \( b_1 \) or \( b_2 \), or both. Note that \( b_2 \) can never be larger than \( b/2 \) and \( 0 < b_1 < b_2 \).

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\(^3\) Poorly designed here mean that it results in excessive stretching.
Special Case Aileron Sizing: Constant Chord Wing

The following expression can be used to determine the spanwise location of the inboard edge of the aileron, for a given outboard edge. It only applies to constant chord (“Hershey bar”) wings.

Physical dimension:

\[
b_1 = \sqrt{\left(\frac{pb}{2V}\right) \frac{C_{ip}}{C_{ls}} \cdot \delta_a b^2 + b_2^2}
\]

\[\text{(7)}\]

Fractional dimension:

\[
b_1 = \sqrt{\left(\frac{pb}{2V}\right) \frac{C_{ip}}{C_{ls}} \cdot \delta_a + \left(\frac{b_1}{b}\right)^2}
\]

\[\text{(8)}\]

Where:

- \(b_1\) = Spanwise station for the inboard edge of the aileron, in ft or m.
- \(b_2\) = Spanwise station for the outboard edge of the aileron, in ft or m.

DERIVATION:

Begin with Equation (1) and solve for \(C_{ls}a\):

\[
\left(\frac{pb}{2V}\right) = -\frac{C_{ls}a}{C_{ip}} \delta_a \quad \Rightarrow \quad C_{ls}a = -\left(\frac{pb}{2V}\right) \frac{C_{ip}}{\delta_a}
\]

\[\text{(i)}\]

Roll authority for a rectangular wing can be shown to be:

\[
C_{ls}a = \left(\frac{pb}{2V}\right) \frac{C_{ip}}{\delta_a} = \frac{c_{ls}a (b_2^2 - b_1^2)}{b^2}
\]

\[\text{(ii)}\]

Since our target is to determine the inboard station, \(b_1\), for the aileron we solve for it using Equation (ii):

\[
b_1 = \sqrt{\left(\frac{pb}{2V}\right) \frac{C_{ip}}{\delta_a} b^2 + b_2^2}
\]

\[\text{(iii)}\]

QED

Figure 3: Definition of the aileron geometry.
EXAMPLE 1:

A UAV is being designed with a Hershey bar wing whose dimensions are shown in Figure 4. The maximum aileron ground deflection is 20°. Assuming that 75% of that will be achievable in flight, determine the roll rate for maximum aileron deflection at V = 100 KTAS if change in lift with aileron deflection, $c_{la}$, has been found to equal 3.165 per radian and the airfoil’s lift curve slope is $c_{la} = 5.322$ per radian and section drag coefficient is taken as $c_{do} = 0.010$. Compare the results to that obtained from the Vortex-Lattice code SURFACES presented in the same examples.

![Figure 4: Example geometry.](image)

SOLUTION:

Determine the derivative $C_{la}$

$$
C_{la} = \frac{dC_l}{d\delta_a} = \frac{2c_{la} \cdot c}{Sb} \int_{b/2}^{b} y \cdot dy = \frac{2(3.165)}{12 \times 12} \left[ \frac{y^2}{2} \right]_2^6 = 0.5936 / \text{rad} \quad (= 0.01036 / ^\circ)
$$

STEP 2: Determine the derivative $C_p$

$$
C_p = -\frac{c_{la} + c_{do}}{6} = -\frac{5.322 + 0.010}{6} = -0.8887 / \text{rad}
$$

STEP 3: Determine the roll helix angle

Based on this the roll rate at 100 KTAS can be found from Equation (1), where the maximum achievable deflection amounts to $20^\circ \times 0.75 = 15^\circ$:

$$
\frac{pb}{2V} = -\frac{C_{la} \delta_a}{C_{p}} = -\frac{0.5936}{-0.8887 \left(15^\circ \frac{\pi}{180} \right)} = 10.02 \text{ deg}
$$

STEP 4: Determine the roll rate $p$

$$
p = \frac{C_{la} \delta_a}{C_{p}} \left( \frac{2V}{b} \right) = \frac{0.5936}{-0.8887 \left(15^\circ \frac{2 \times 168.8}{12} \right)} = 282.9 \text{ ^\circ/s}
$$

A model of this wing was constructed in SURFACES. Once complete, the Tasks->Stability Derivatives... was selected on the VLM Console and the two options checked as shown in the image below. The aileron authority and roll damping where then determined and found to equal:
\[ C_{l_{\alpha}} = 0.4134 / \text{rad} \quad \text{and} \quad C_{l_p} = -0.6336 / \text{rad} \]

Therefore, SURFACES predicts the following roll rate for the wing:

\[ p = -\frac{C_{l_{\alpha}}}{C_{l_p}} \delta_a \left( \frac{2V}{b} \right) = -\frac{0.4134}{-0.6336} \left( 15^\circ \frac{2 \times 168.8}{12} \right) = 275.3 \, ^\circ / \text{s} \]

As can be seen, the solution from SURFACES accounts for tip effects both for the root and tip of the aileron, as well as that of the wing. That interaction is quite complicated and it is an important detail to capture so that one does not overestimate the roll-rate.
Maximizing Responsiveness

Some airplanes require flap span to be maximized to meet requirements for stall speed. This means that the aileron span is less than ideal. The designer can attempt to improve the effectiveness of the remaining ailerons by positioning the centroid of the aileron planform as close as possible to the location where their spanwise section moment reaches maximum.

Consider the tapered wing planform (halfspan) below:

![Diagram of wing planform](image)

**Figure 5: Definition of an infinitesimal segment ΔS on the wing.**

Spanwise section moment (analogous to section lift or section lift coefficient) is defined as follows:

\[ ΔC_m = y \cdot C_l \quad ↔ \quad ΔM_x = y \cdot (q \cdot C_l \cdot ΔS) \]

Where;
- \( ΔC_m \) = Spanwise moment coefficient
- \( C_l \) = Section lift coefficient
- \( ΔM_x \) = Elemental rolling moment
- \( y \) = Spanwise station
- \( q \) = Dynamic pressure
- \( ΔS \) = Area of elemental strip

Consider the wing shown below, which show the distribution of section lift coefficients:
Figure 6: Distribution of section lift coefficient along the wing⁴.

Let’s zoom in and show the section moments for a strip:

Figure 7: Generation of spanwise section moment.

Let’s plot the section moments for each strip.

⁴ Generated with the Vortex-Lattice code SURFACES.
Figure 8: Distribution of section moments along the wing.

The effect of deflecting ailerons on the distribution of section lift coefficients can be seen in Figure 9 below. The ailerons are deflected some 15° and the wing’s AOA amounts to 8° at 100 KCAS.

Figure 9: Typical impact of deflecting ailerons on the section lift coefficients\(^5\).

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\(^5\) Generated with the Vortex-Lattice code SURFACES.
Figure 10: Impact of aileron deflection on the flow field behind the aircraft. Note the difference in the wing tip vortices\(^6\).

### Aileron Stick Forces

In a conventional, human operated aileron control system, the pilot must react the hinge moment the results from deflecting the control surface. This is done by applying a force to the proper control (a stick, yoke, or a "steering wheel").

![Hinge Moment Diagram](image)

**Figure 11:** The hinge moment (HM) is reacted as a force, either directly by the pilot or by a control system (typically hydraulic or electric).

Hinge moments are highly affected by the geometry of the controls, including the hinge location and shape of the control. A general expression for the hinge moment is given below:

\[
HM = \frac{1}{2} \rho V^2 C_f S_f C_h
\]  

(9)

Where;  
- \(C_f\) = Flap chord (aft of hingeline)  
- \(C_h\) = Hinge moment

\(^6\) Generated with the Vortex-Lattice code SURFACES.
$S_f = \text{Flap area (aft of hingeline)}$

$V = \text{Airspeed,}$

$\rho = \text{Density of air}$

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The hinge moment coefficient is given by:

$$C_h = C_{h0} + C_{h\alpha} \cdot \alpha + C_{h\delta} \cdot \delta + C_{h\delta_s} \cdot \delta_s$$

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**WHAT IS ProAdvice?**

Pro-Advices are short and simplified excerpts from Professor Gudmundsson’s design handbook *Aircraft Preliminary Design Handbook* and are intended to provide the aircraft designer with clear and concise analysis methods for the aircraft designer. This handbook is currently in development. Snorri Gudmundsson is an Assistant Professor of Aerospace Engineering at Embry-Riddle Aeronautical University in Daytona Beach, Florida, where he teaches Aircraft Preliminary Design to senior engineering students.